Berufliches Gymnasium
Übungsaufgaben Ableitungen

BERUFSBILDENDE SCHULEN
„OTTO VON GUERICKE"
MAGDEBURG

Bestimmen Sie jeweils die erste Ableitung der folgenden Funktionen unter Verwendung der Kettenregel.

(a) $f(x) = (x-1)^2$

(b) $f(x) = (x^2+2x)^5$

(c) $f(x) = \left(\frac{1}{x}+x\right)^{-1}$

(d) $f(x) = (x-1)^2(x+1)^{-2}$

(e) $f(x) = \sin\left(\frac{3}{2}x^2\right)$

(f) $f(x) = (x-1)^4 \cdot \frac{1}{\cos(x)}$

(g) $f(x) = \left(\frac{1}{4}x^4 - \sin(x)\right)^2$

(h) $f(x) = 1 + x^2 \cdot e^{x^2}$

(i) $f(x) = 3 \cdot \cos(\cos(x)) \cdot e^x$

(j) $f(x) = \frac{1}{(2x-5)^7} + x - 2$

(k) $f(x) = e^{\sqrt{x}} + \frac{1}{e^{\sqrt{x}}} - 1$

(l) $f(x) = \sqrt[3]{\sin(x)} + e^x$

(m) $f(x) = \sqrt[3]{2x+12}$

(n) $f(x) = e^x - \frac{1}{\sqrt[3]{27+x}}$

Bestimmen Sie jeweils die erste Ableitung der folgenden Funktionen unter Verwendung der Kettenregel.

(a) $f(x) = (x-1)^2$

$$f'(x) = 2 \cdot (x-1)$$

(b) $f(x) = (x^2 + 2x)^5$

$$f'(x) = 5 \cdot (x^2 + 2x)^4 \cdot (2x + 2)$$

(c) $f(x) = \left(\frac{1}{x} + x\right)^{-1}$

$$f'(x) = -\left(\frac{1}{x} + x\right)^{-2} \cdot \left(1 - \frac{1}{x^2}\right)$$

(d) $f(x) = (x-1)^2(x+1)^{-2}$

$$f'(x) = 2(x-1) \cdot (x+1)^{-2} - 2(x-1)^2(x+1)^{-3}$$

$$f'(x) = \frac{2(x-1)}{(x+1)^2} \cdot \left(1 - \frac{x-1}{x+1}\right)$$

(e) $f(x) = \sin\left(\frac{3}{2}x^2\right)$

$$f'(x) = \cos\left(\frac{3}{2}x^2\right) \cdot 3x = 3x \cdot \cos\left(\frac{3}{2}x^2\right)$$

(f) $f(x) = (x-1)^4 \cdot \frac{1}{\cos(x)}$

$$f(x) = (x-1)^4(\cos(x))^{-1}$$

$$f'(x) = 4(x-1)^3(\cos(x))^{-1} + (x-1)^4(\cos(x))^{-2} \cdot \sin(x)$$

$$f'(x) = \frac{4(x-1)^3}{\cos(x)} + \frac{(x-1)^4 \cdot \sin(x)}{\cos^2(x)}$$

(g) $f(x) = \left(\frac{1}{4}x^4 - \sin(x)\right)^2$

$$f'(x) = 2\left(\frac{1}{4}x^4 - \sin(x)\right) \cdot (x^3 - \cos(x))$$

(h) $f(x) = 1 + x^2 \cdot e^{x^2}$

$$f'(x) = 2xe^{x^2} + 2x \cdot x^2 e^{x^2} = 2xe^{x^2}(1 + x^2)$$

(i) $f(x) = 3 \cdot \cos(\cos(x)) \cdot e^x$

$$f'(x) = 3\sin(\cos(x)) \cdot \sin(x)\, e^x + 3\cos(\cos(x))\, e^x$$

$$f'(x) = 3e^x(\sin(\cos(x)) \cdot \sin(x) + \cos(\cos(x)))$$

(j) $f(x) = \frac{1}{(2x-5)^7} + x - 2 = (2x-5)^{-7} + x - 2$

$$f'(x) = -7(2x-5)^{-8} \cdot 2 + 1 = 1 - \frac{14}{(2x-5)^8}$$

(k) $f(x) = e^{\sqrt{x}} + \frac{1}{e^{\sqrt{x}}} - 1 = e^{x^{\frac{1}{2}}} + e^{-x^{\frac{1}{2}}} - 1$

$$f'(x) = \frac{1}{2}x^{-\frac{1}{2}}e^{x^{\frac{1}{2}}} - \frac{1}{2}x^{-\frac{1}{2}}e^{-x^{\frac{1}{2}}} = \frac{e^{\sqrt{x}}}{2\sqrt{x}} - \frac{1}{2e^{\sqrt{x}} \cdot \sqrt{x}}$$

(l) $f(x) = \sqrt[3]{\sin(x)} + e^x = (\sin(x))^{\frac{1}{3}} + e^x$

$$f'(x) = \frac{1}{3}(\sin(x))^{-\frac{2}{3}} \cdot \cos(x) + e^x = \frac{\cos(x)}{3 \cdot \sqrt[3]{\sin^2(x)}} + e^x$$

(m) $f(x) = \sqrt[3]{2x + 12} = (2x + 12)^{\frac{1}{3}}$

$$f'(x) = \frac{1}{3}(2x + 12)^{-\frac{2}{3}} \cdot 2 = \frac{2}{3\sqrt[3]{(2x + 12)^2}}$$

(n) $f(x) = e^x - \frac{1}{\sqrt[3]{27+x}} = e^x - (27 + x)^{-\frac{1}{3}}$

$$f'(x) = e^x + \frac{1}{3}(27 + x)^{-\frac{4}{3}} = e^x + \frac{1}{3\sqrt[3]{(27 + x)^4}}$$